PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

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Phil. Trans. R. Soc. Lond. A 1989 **328**, 351-363
doi: 10.1098/rsta.1989.0040

Phil. Trans. R. Soc. Lond. A **328**, 351–363 (1989)
Printed in Great Britain

Fluctuations in the Earth's rotation and the topography of the core–mantle interface

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As arguments in favour of the notion that very slow convection in the highly viscous mantle is confined to the upper 700 km gradually weakened over the past 20 years, so geophysicists have increased their willingness to entertain the idea that significant horizontal variations in temperature and other structural parameters occur at all levels in the lower mantle. Concomitant density variations, including those caused by distortions in the shape of the core–mantle interface, would contribute substantially to long-wavelength features of the Earth's gravity field and also affect seismic travel times. The implied departures from axial symmetry in the thermal and mechanical boundary conditions thus imposed by deep mantle convection on the underlying low-viscosity liquid metallic core would affect not only *spatial* variations in the long-wavelength features of the main geomagnetic field (which is generated by dynamo action involving comparatively rapid chaotic magnetohydrodynamic flow in the core) but also *temporal* variations on all relevant timescales, from decades and centuries characteristic of the geomagnetic secular variation to tens of millions of years characteristic of changes in the frequency of polarity reversals.

Core motions should influence the rotation of the 'solid' Earth (mantle, crust and cryosphere), and in the absence of any quantitatively reasonable alternative line of attack, geophysicists have long supposed that irregular 'decade' fluctuations in the length of the day of about 5×10^{-3} s must be manifestations of angular momentum exchange between the core and mantle produced by time-varying torques at the core–mantle interface. The stresses responsible for these torques comprise (*a*) *tangential* stresses produced by viscous forces in the thin Ekman–Hartmann boundary layer just below the interface and also by Lorentz forces associated with the interaction of electric currents in the weakly conducting lower mantle with the magnetic field there, and (*b*) *normal* stresses produced largely by dynamical pressure forces acting on irregular interface topography (i.e. departures in shape from axial symmetry). The hypothesis that topographic stresses might provide the main contribution to the torque was introduced by the author in the 1960s and the present paper gives details of his recently proposed method for using Earth rotation and other geophysical data in a new test of the hypothesis. The method provides a scheme for investigating the consistency of the hypothesis with various combinations of 'models' of (*a*) motions in the outer reaches of the core based on geomagnetic secular variation data, and (*b*) core–mantle interface topography based on gravity and seismic data, thereby elucidating the validity of underlying assumptions about the dynamics and structure of the Earth's deep interior upon which the various 'models' are based. The scheme is now being applied in a complementary study carried out in collaboration with R. W. Clayton, B. H. Hager, M. A. Spieth and C. V. Voorhies.

1. Introduction

Recent advances in seismic tomography and the study of mantle convection are influencing progress in other areas of geophysics and geochemistry. Of particular interest in connection

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with the new international study of the Earth's deep interior (SEDI) now being organized under the auspices of the International Union of Geodesy and Geophysics will be improvements in our knowledge of the structure, composition and dynamics of the lower mantle and the liquid metallic core, where the geomagnetic field is produced by the self-exciting magnetohydrodynamic dynamo process. Historically, seismology has provided Earth scientists with crucial detailed knowledge about the radial variation of the Earth's density and elastic properties when averaged over spherical surfaces, but without having had much to say until fairly recently about asymmetric features of the structure of the Earth's deep interior (see Doornbos 1988; Silver & Carlson 1988; Woodhouse & Dziewonski, this Symposium). So those of us who in the 1960s and 1970s had been obliged to invoke such features to make sense of other types of geophysical data were unable then to call upon seismologists for detailed guidance. When, for instance, I was preparing a discussion (Hide 1970) of possible geophysical implications of observations of long-wavelength features of the Earth's main magnetic and gravitational fields and of the so-called 'decade' variations in the Earth's rate of rotation, my conclusions could not usefully be checked against seismic data. (An attempt on my part to use the inadequate seismic data then available to test the validity of hypothetical maps of core–mantle interface topography based on gravity data alone (Hide & Horai 1968) was amateurish, short-lived and thoroughly unsuccessful (but see Vogel 1960).)

I was viewing (so to speak) the Earth's interior through the eyes of a fluid dynamicist with some knowledge of the hydrodynamics and magnetohydrodynamics (MHD) of rapidly rotating fluids combined with a long-standing interest in the dynamo theory of the origin of the main geomagnetic field and its bearing on the interpretation of various properties of that field and its secular changes, including the striking discovery by palaeomagnetic workers that polarity reversals vary markedly in frequency over geological time (for references, see Jacobs 1975, 1984; Runcorn *et al.* 1982; Merrill & McElhinny 1983). In all realistic theoretical models, the specification of boundary conditions is crucial, and no thoroughly satisfactory model of the geodynamo can be formulated without due attention being paid to the thermal, mechanical and electromagnetic boundary conditions under which the full set of MHD equations would have to be solved. Much of course can be learned about dynamos through the study of highly simplified models (for references, see Moffatt 1978*a*; Jacobs 1987) but it was evident that essential features of a model capable of accounting in detail for the principal spatial and temporal characteristics of the main field as revealed by hard-won geomagnetic and palaeomagnetic data would be small but dynamically significant irregular departures from spherical symmetry in the boundary conditions.

To anyone who had thought seriously about these matters it seemed likely that irregular aspherical features of the shape of the core–mantle interface and the thermal field there could be produced and modulated over geological timescales by slow and time-varying convection in the highly viscous lower mantle. Having got off to a slow start, the idea is becoming accepted as providing a useful working hypothesis or paradigm, as several contributions to this very timely Discussion Meeting will attest and provide detailed references. It will be interesting to see what consensus emerges from all these contributions, and even more important will be our attempts to identify the crucial questions upon which attention must be focused in the immediate future. Recent progress made with the direct use of gravity, magnetic and seismic data will be discussed by other speakers (see papers by Hager & Richards, Gubbins, Woodhouse & Dziewonski, all in this Symposium). It is my allotted task to outline some of the implications for the Earth's deep interior of recent work on angular momentum transfer between the core and mantle, as evinced largely by the irregular variations in the length of the day that would be left when angular momentum exchange between the 'solid Earth' (mantle, crust and cryosphere) and overlying atmosphere and hydrosphere, moment of inertia changes due to internal processes, and various other effects had been allowed for (see §2 below).

A mechanical couple or torque acting between two bodies transfers angular momentum from one to the other at a rate proportional to the strength of the couple. The couple – a vector quantity with a precise definition – is caused by physical interactions of various kinds between the bodies, but there can obviously be other interactions that do not give rise to a mechanical couple. Now the imprecise term 'core–mantle coupling', usually qualified by the adjective 'viscous', 'electromagnetic' or 'topographic' (see equation (2.1)), is widely used in the extensive literature on the interpretation of the decade variations in the length of the day as being manifestations of angular momentum exchange between the core and mantle (but the more precise term 'torquing' is seen only occasionally). This usage turns out to be unfortunate, for it may have led to the misunderstandings evident in the writings of authors who, by interpreting the term 'coupling' as embracing a wider class of interactions than just those that directly produce mechanical 'couples', have promoted a false dichotomy concerning the relative importance of thermal and topographic contributions to interactions between the core and mantle. Deep convection in the highly viscous mantle would produce substantial horizontal temperature variations near the core–mantle interface, and concomitant stresses in the mantle would deform the core–mantle interface, thereby producing irregular aspherical topography of the interface. Through their effects on motions in the fluid core, these fields of temperature and topography at the bottom of the mantle would affect the behaviour of the main geomagnetic field on all relevant timescales, from those characteristic of the geomagnetic secular variation (decades and centuries) to those associated with mantle convection and geological processes (millions of years and longer).

Detailed theoretical studies of these interactions will be of great interest in geophysics, just as investigations of both mean *and* time-varying distortions of atmospheric motions due to the presence of mountains and imposed irregular horizontal temperature variations at the Earth's surface are important in dynamical meteorology (see e.g. Kalnay & Mo 1986). Unlike the field of topography at the core–mantle interface, however, the associated temperature field in the lower mantle cannot contribute directly to the mechanical couple acting at the interface, although together with the ambient magnetic field, they are likely to produce important indirect effects. In my own work on decade variations in the length of the day I introduced the idea and emphasized the likely importance of topographic contributions to that couple (Hide 1969). But I have never dismissed thermal interactions between the mantle and core as being geophysically unimportant; on the contrary, the first discussion of the likely significance of such interactions was possibly given in a paper of mine (Hide 1967; see also Hide & Malin 1971; Malin & Hide 1982; Bloxham & Gubbins 1987). So I take this opportunity to reiterate my long-held view that the investigation of the dynamical processes involved in these various interactions and of their geophysical consequences, as in the important work reviewed by Gubbins (this Symposium), deserves high priority in theoretical work on the structure and dynamics of the Earth's deep interior.

2. Torques at the core–mantle interface

Irregular fluctuations in the rate of rotation of the solid Earth and also in the alignment of the rotation axis relative to the figure axis are produced by two principal agencies. These are: (*a*) changes in the inertia tensor of the solid Earth associated with the melting of ice, mantle convection, earthquakes, deformations associated with a variety of applied stresses, etc.; and (*b*) angular momentum transfer between the solid Earth and the fluid regions (liquid core, hydrosphere and atmosphere) with which the solid Earth is in contact. Of direct relevance in the investigation of the structure and dynamics of the liquid metallic core and lower mantle are the so-called 'decade' fluctuations in the length of the day (LOD) of up to about 5×10^{-3} s on timescales upwards of a few years and associated polar motion. The atmosphere is now known to be responsible for observed LOD fluctuations of about 1×10^{-3} s on timescales ranging from a few days to a few years and also for much if not all the observed polar motion on these timescales. But in the face of quantitative difficulties involved in accounting for the comparatively large but slower 'decade' fluctuations in terms of atmospheric (or oceanic) processes or of plausible changes in the inertia tensor of the solid Earth on relevant timescales, geophysicists have argued that the fluctuations are best regarded as manifestations of angular momentum transfer between the core and mantle, caused by dynamical stresses exerted on the mantle by irregular ('chaotic') flow in the core with typical speeds of about 3×10^{-4} m s^{-1} (for extensive references, see Munk & Macdonald 1960; Hide 1977, 1984, 1985; Lambeck 1980; Rochester 1984; Melchior 1986; Cazenave 1986; Dickey *et al.* 1986; Chao & Gross 1987; Hinderer *et al.* 1987; Moritz & Mueller 1987; Jault *et al.* 1988; Merriam 1988; Vondrák & Pejović 1988; Wahr 1988).

When integrated over the whole of the core–mantle boundary (CMB), these stresses give rise to a net couple

$$\boldsymbol{L}^*(t) \equiv \iint_{\text{CMB}} \boldsymbol{r} \times [\boldsymbol{F}_\text{V} + \boldsymbol{F}_\text{E} + \boldsymbol{F}_\text{T}]\, \mathrm{d}A. \tag{2.1}$$

Here t denotes time, $\boldsymbol{r}$ is the position vector of a general point referred to the Earth's centre of mass, $\mathrm{d}A$ the area element of the CMB, $\boldsymbol{F}_\text{V}(t)$ the stress associated with shearing motions in the viscous boundary layer, $\boldsymbol{F}_\text{E}(t)$ the electromagnetic stress associated with the Lorentz force $\boldsymbol{j} \times \boldsymbol{B}$ (where $\boldsymbol{j}$ is the electric current density and $\boldsymbol{B}$ the magnetic field) and $\boldsymbol{F}_\text{T}(t)$ the 'topographic stress' due to the action of normal pressure forces on bumps (including the equatorial bulge) in the shape of the core–mantle interface. The 'axial' component of $\boldsymbol{L}^*$ produces changes in the length of the day; the 'equatorial components' move the pole of rotation of the solid Earth relative to its axis of figure (see equation (2.9) below).

The viscous contribution $\boldsymbol{F}_\text{V}$ would be negligible on all but the most extreme assumptions about viscous forces in the core (see Bullard *et al.* 1950). Much more promising but still controversial owing to both qualitative and quantitative difficulties is the electromagnetic contribution, which has been the subject of a considerable number of theoretical studies since it was first considered by Bullard *et al.* (1950) (for references, see Rochester 1984; Paulus & Stix 1986; Roberts 1989). The theory of topographic coupling is less straightforward (Hide 1969, 1977; Anufriev & Braginsky 1977; Eltayeb & Hassan 1979; Moffatt 1978*b*; Roberts 1989), but rough dynamical arguments show that bumps no more than 10^3 m in vertical amplitude might suffice to make the axial component of $\boldsymbol{L}^*(t)$ large enough to account for the magnitude of the observed decade LOD changes (Hide 1969).

The determination of the topographic contribution $\boldsymbol{L}(t)$ (say) to $\boldsymbol{L}^*(t)$ (see equation (2.1)) from geophysical data using a method proposed by Hide (1986) is currently the subject of a complementary study (see §4 below). $\boldsymbol{L}(t)$ is given by the equation

$$\boldsymbol{L}(t) \equiv \iint_{\text{CMB}} \boldsymbol{r} \times \boldsymbol{F}_{\text{T}} \, \mathrm{d}A = \iint_{\text{CMB}} \boldsymbol{r} \times p_{\text{s}} \boldsymbol{n} \, \mathrm{d}A, \tag{2.2}$$

where $\boldsymbol{n}$ is the outwardly directed unit vector normal to the core–mantle boundary and p_{s} is the dynamic pressure associated with core motions just below the CMB. If the CMB is the locus of points where

$$r = r(\theta, \phi) = c + h(\theta, \phi), \tag{2.3}$$

(where c is the mean radius of the core (3480 km), θ co-latitude and ϕ longitude) then

$$\boldsymbol{L} = -c^2 \int_0^{2\pi} \int_0^{\pi} (\boldsymbol{r} \times p_{\text{s}} \nabla_{\text{s}} h) \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi, \tag{2.4}$$

where $\nabla_{\text{s}} \equiv c^{-1}(\hat{\boldsymbol{\theta}}\,\partial/\partial\theta + \hat{\boldsymbol{\phi}} \operatorname{cosec}\theta\,\partial/\partial\phi)$ if $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{\phi}}$ are unit vectors in the directions of increasing θ and ϕ respectively. Now it is readily shown that the surface integral of $\boldsymbol{r} \times \nabla_{\text{s}}(hp_{\text{s}})$ vanishes, so that equation (2.4) can be expressed in the equivalent and more convenient form

$$\boldsymbol{L}(t) = c^2 \int_0^{2\pi} \int_0^{\pi} (\boldsymbol{r} \times h \nabla_{\text{s}} p_{\text{s}}) \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi. \tag{2.5}$$

An expression for $h\nabla_{\text{s}} p_{\text{s}}$ is obtainable by considering the hydrodynamical equation of motion, which throughout most of the core can be approximated by

$$2\rho \bar{\boldsymbol{\Omega}} \times \boldsymbol{u} + \nabla p - \boldsymbol{g}\rho \approx \boldsymbol{j} \times \boldsymbol{B}, \tag{2.6}$$

where ρ denotes density, p the total pressure, $\boldsymbol{u}$ the eulerian flow velocity in a frame of reference that rotates with angular velocity $\bar{\boldsymbol{\Omega}}$ relative to an inertial frame, and $\boldsymbol{g}$ is the acceleration due to gravity and centrifugal effects (see Hide 1986, equation (2.1)). It has long been recognized that over length scales comparable with the radius of the core, Coriolis effects on fluid motions there, as represented by the term $2\rho\boldsymbol{\Omega} \times \boldsymbol{u}$ in equation (2.6), are many orders of magnitude greater than the neglected relative acceleration terms $\rho\,\partial\boldsymbol{u}/\partial t$ and $\rho(\boldsymbol{u}\cdot\nabla)\,\boldsymbol{u}$ and the viscous term. The magnitude of the Lorentz term $\boldsymbol{j} \times \boldsymbol{B}$ on the right-hand side of equation (2.6) is uncertain, for we do not know the strength of the magnetic field within the core of the Earth. It has been argued that $\boldsymbol{B}$ is unlikely on average to exceed that value for which 'magnetostrophic' balance obtains, when $2\rho\bar{\boldsymbol{\Omega}} \times \boldsymbol{u}$ and $\boldsymbol{j} \times \boldsymbol{B}$ have the same order of magnitude (see Hide & Roberts 1978). Moreover, it is a fortunate circumstance that owing to the relatively poor electrical conductivity of the mantle the magnitudes of $\boldsymbol{j}$ and $\boldsymbol{B}$ near the core–mantle interface will be much less, by about a factor of 10, than the magnetostrophic value, the corresponding magnitude of the right-hand side of equation (2.6) being about 10^{-2} times that of $2\rho\bar{\boldsymbol{\Omega}} \times \boldsymbol{u}$. This implies that motions in the outer reaches of the core (but *not* within the viscous boundary layer of no more than about 1 m in thickness just below the CMB) may be characterized by geostrophic balance between Coriolis acceleration and the dynamic pressure gradient (see Le Mouël 1984), that is to say

$$2\rho \bar{\boldsymbol{\Omega}} \times \boldsymbol{u} \approx -\nabla p + \boldsymbol{g}\rho, \tag{2.7}$$

the curl of which gives $(2\bar{\boldsymbol{\Omega}}\cdot\nabla)\,\boldsymbol{u} \approx \rho^{-2}\nabla\rho\times\nabla p \approx \rho^{-1}\boldsymbol{g}\times\nabla\rho$. Taking $|2\bar{\boldsymbol{\Omega}}| \approx 10^{-4}\,\mathrm{s}^{-1}$, $\rho \approx 10^4\,\mathrm{kg\,m^{-3}}$ and $|\boldsymbol{u}| \approx 3\times10^{-4}\,\mathrm{m\,s^{-1}}$ gives $|\nabla p - \boldsymbol{g}\rho| \approx 3\times10^{-4}\,\mathrm{N\,m^{-3}}$, corresponding to pressure variations of about $300\,\mathrm{N\,m^{-2}}$ (3 mbar) over horizontal distances of the order of 10^6 m and to concomitant seismologically undetectable but dynamically highly significant fractional density and pressure variations of order $|2\bar{\boldsymbol{\Omega}}\times\boldsymbol{u}|/|\boldsymbol{g}| = 3\times10^{-9}$!

Equations (2.7) and (2.5) provide the basis of the above-mentioned method for evaluating the torque on the mantle due to topographic stresses at the CMB. Denote by $\boldsymbol{u}_s$ the eulerian flow velocity in the free stream just below the viscous boundary layer at the CMB, and by (u_s, v_s, w_s) the (r, θ, ϕ) components of $\boldsymbol{u}_s$, where u_s is typically so much smaller in magnitude than v_s and w_s that it can safely be set equal to zero. If $\nabla_s p_s$ is the corresponding value of the horizontal pressure gradient and $\bar{\rho}_s$ the horizontally averaged value of $\rho(r \approx c)$ then by equation (2.7) we have $(v_s, w_s) = (2\bar{\rho}\bar{\Omega}c\cos\theta)^{-1}\,(-\operatorname{cosec}\theta\,\partial p_s/\partial\phi, \partial p_s/\partial\theta)$, so that v_s and $\partial p_s/\partial\theta$ vanish on the Equator.

By equations (2.5) and (2.7),

$$\boldsymbol{L}(t) = 2\bar{\rho}_s\Omega c^3 \int_0^{2\pi}\!\int_0^{\pi} h(\theta,\phi)\,\boldsymbol{u}_s(\theta,\phi,t)\sin\theta\cos\theta\,\mathrm{d}\theta\,\mathrm{d}\phi \tag{2.8}$$

where on the timescales of interest here, $\boldsymbol{u}_s$ depends on t but h does not. Consider a set of body-fixed axes x_i $(i = 1, 2, 3)$ aligned with the principal axes of the solid Earth and rotating with angular velocity $\omega_i(t)$ about its centre of mass. By equation (2.8), the topographic torque $\boldsymbol{L}(t)$ has three components $L_i(t)$ $(i = 1, 2, 3)$ in this system given by

$$L_i(t) = 2\bar{\rho}_s\bar{\Omega}c^3 \int_0^{2\pi}\!\int_0^{\pi} h\{v_s\cos\theta\cos\phi - w_s\sin\phi,\, v_s\cos\theta\sin\phi + w_s\cos\phi,\, -v_s\sin\theta\}\sin\theta\cos\theta\,\mathrm{d}\theta\,\mathrm{d}\phi. \tag{2.9}$$

The axial component L_3 changes the length of the day $(2\pi/\omega_3)$ and the equatorial components L_1 and L_2 move the pole of rotation, whose position relative to the figure axis is specified by ω_1 and ω_2 (see equations (3.36)–(3.39) below).

The dynamical processes that produce within the fluid regions of the Earth the stresses responsible for transferring angular momentum to the solid Earth are by no means fully understood. Their elucidation will require much further study by theoreticians and experimentalists concerned with the hydrodynamics and magnetohydrodynamics of rotating fluids. Discussion of the fascinating problems involved lies beyond the scope of the present paper (see Hide 1969, 1977; Anufriev & Braginsky 1977; Moffatt 1978*b*; Eltayeb & Hassan 1979; Roberts 1989), but a general observation on the strategy to be followed in this line of research is worth making. It is important not to oversimplify models of the interaction of core motions with the core–mantle interface, and in this connection, the case when effects due to nonlinear advection of momentum, buoyancy forces, magnetic fields and viscosity are all neglected is instructive. Then, in accordance with well-known results concerning the 'spin-up' of fluids in irregular containers (see, for example, Greenspan 1968), the dynamical pressure field would be symmetric about the equatorial plane and vary with θ and ϕ in such a way that the L_3 vanishes! This is understandable, for when angular momentum is exchanged between the solid Earth and its fluid regions, the total rotational kinetic energy of the whole system must change if the speed of rotation of the solid Earth changes, even if the total angular momentum of the whole system remains constant. So successful models of the interaction processes must

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include mechanisms capable of producing transformations between rotational kinetic energy and other forms of energy (non-rotational kinetic energy, gravitational potential energy, magnetic energy and thermal energy) through the action of some or all the above-mentioned agencies.

3. Dynamics of the Earth's rotation

In this brief account of non-rigid body rotation, the equations needed for the study of the variable rotation of the Earth are derived in a form appropriate to this discussion of core–mantle interactions. Euler's dynamical equations describing the response of the 'whole Earth' (namely the 'solid Earth' plus the underlying inner (solid) core and outer (liquid) core and the overlying hydrosphere and lower and upper atmosphere) to an externally applied torque $\hat{L}_i$ are the following:

$$\mathrm{d}H_i/\mathrm{d}t + \epsilon_{ijk}\,\omega_j\,H_k = \hat{L}_i, \tag{3.1}$$

the usual convention being used for repeated suffices. Here ϵ_{ijk} is the alternating tensor (with values 0 or ± 1), $\mathrm{d}/\mathrm{d}t$ is the time-derivative in the rotating frame, and H_i, the absolute angular momentum, is given by

$$H_i(t) \equiv I_{ij}(t)\,\omega_j(t) + h_i(t). \tag{3.2}$$

The quantity I_{ij} is the variable inertia tensor defined by the volume integral (taken over the whole Earth)

$$I_{ij} \equiv \oiiint \rho\,(x_k\,x_k\,\delta_{ij} - x_i\,x_j)\,\mathrm{d}V, \tag{3.3}$$

where $\mathrm{d}V$ denotes volume element, δ_{ij} is the Kronecker delta (with values 0 and 1) and

$$h_i \equiv \oiiint \rho\epsilon_{ijk}\,x_j\,u_k\,\mathrm{d}V \tag{3.4}$$

is the angular momentum due to motion u_i relative to the axes x_i. Substitution in (3.1) gives the Liouville equation

$$\mathrm{d}/\mathrm{d}t(I_{ij}\,\omega_j + h_i) + \epsilon_{ijk}\,\omega_j(I_{kl}\,\omega_l + h_k) = \hat{L}_i, \tag{3.5}$$

further details of which, including expressions for all three components, can be found in various texts (see, for example, Munk & MacDonald 1960; Lambeck 1980).

Because the rotation of the Earth departs only slightly from steady rotation about the polar axis of figure, we write

$$\omega_i = (\omega_1, \omega_2, \omega_3) = \Omega(m_1, m_2, 1 + m_3) \tag{3.6}$$

where Ω is the mean rotational speed of the Earth, 0.7292115×10^{-4} radians per sidereal second. The quantities m_1, m_2 and m_3 are very much less than unity and $|\dot{m}_i| \equiv |\mathrm{d}m_i/\mathrm{d}t| \ll \Omega$. The motion of the pole is given by $(m_1(t), m_2(t))$ and it will be convenient to define the quantity

$$\boldsymbol{m} \equiv m_1 + \mathrm{i}m_2 \tag{3.7}$$

where $\mathrm{i} \equiv \sqrt{-1}$. If $\Delta\Lambda(t) \equiv \Lambda(t) - \Lambda_0$, the difference between the instantaneous length of the day $2\pi/\omega_3$ and its average value $2\pi/\Omega$, then by equation (3.6),

$$m_3(t) = -\Delta\Lambda(t)/\Lambda_0. \tag{3.8}$$

In the same spirit, we now treat the quantities I_{ij} and h_i. Define

$$I_{ij} \equiv I_{ij}^{(c)} + I_{ij}^{(m)} + I_{ij}^{(a)}, \tag{3.9}$$

$$h_i \equiv h_i^{(c)} + h_i^{(m)} + h_i^{(a)}, \tag{3.10}$$

$$A \equiv A^{(c)} + A^{(m)} + A^{(a)}, \tag{3.11}$$

and

$$C \equiv C^{(c)} + C^{(m)} + C^{(a)}, \tag{3.12}$$

where the superscripts (c), (m) and (a) refer respectively to the whole core (inner and outer), the 'solid Earth' (mantle, crust and cryosphere), and the outer fluid layers (comprising the hydrosphere and the upper and lower atmosphere). $A^{(c)}, C^{(c)}$, etc., as defined by equations (3.11) and (3.12) are the principal moments of inertia of the regions to which they refer. It is readily shown by considering the orders of magnitude of the various terms in these equations that

$$C - A \approx 3 \times 10^{-3}\, C \ll C; \quad A^{(a)} \approx C^{(a)} \approx 3 \times 10^{-4}\, A \ll A;$$

$$A^{(c)} \approx C^{(c)} \approx 0.1\, C^{(m)} \ll C^{(m)}; \quad |h_i^{(c)}| \gtrsim |h_i^{(a)}| \gg h_i^{(m)}$$

and

$$|h_i^{(c)} + h_i^{(a)}| \approx 5 \times 10^{-8}\, \Omega C \ll \Omega C. \tag{3.13}$$

So we can write

$$h_i = h_i^{(c)} + h_i^{(a)}, \tag{3.14}$$

$$A = A^{(c)} + A^{(m)}, \quad C = C^{(c)} + C^{(m)} \tag{3.15}$$

and

$$I_{ij} = \begin{pmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & C \end{pmatrix} + \Delta I_{ij}, \tag{3.16}$$

with corresponding separate expressions for $I_{ij}^{(c)}$ and $I_{ij}^{(m)}$, if

$$\Delta I_{ij} \equiv \Delta I_{ij}^{(c)} + \Delta I_{ij}^{(m)} + I_{ij}^{(a)}. \tag{3.17}$$

Adopting a perturbation approach by combining equations (3.6) and (3.16) with equation (3.5) and neglecting second-order quantities, we find

$$\sigma_r^{-1}\, \dot{m}_1 + m_2 = \psi_2, \tag{3.18}$$

$$\sigma_r^{-1}\, \dot{m}_2 - m_1 = -\psi_1, \tag{3.19}$$

$$\dot{m}_3 = \dot{\psi}_3, \tag{3.20}$$

where

$$\sigma_r \equiv \Omega(C-A)/A \tag{3.21}$$

is the 'rigid body' frequency corresponding to a period of about 10 months. The non-homogeneous forcing terms are known as the excitation functions given by

$$\psi_1 \equiv [\Omega^2 \Delta I_{13} + \Omega \Delta \dot{I}_{23} + \Omega h_1 + \dot{h}_2 - \hat{L}_2]/\Omega^2 (C-A), \tag{3.22}$$

$$\psi_2 \equiv [\Omega^2 \Delta I_{23} - \Omega \Delta \dot{I}_{13} + \Omega h_2 - \dot{h}_1 + \hat{L}_1]/\Omega^2 (C-A), \tag{3.23}$$

and

$$\psi_3 \equiv \left[\frac{-\Omega \Delta I_{33} - h_3}{\Omega C}\right] + \frac{1}{\Omega C} \int_0^t \hat{L}_3(\tau)\, d\tau, \tag{3.24}$$

where τ is a dummy variable.

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The solution of equations (3.18) and (3.19) for the equatorial components m_1 and m_2 is

$$\boldsymbol{m}(t) = \exp(\mathrm{i}\sigma_r t)\left\{\boldsymbol{m}(0) - \mathrm{i}\sigma_r \int_0^t \boldsymbol{\psi}(\tau)\exp(-\mathrm{i}\sigma_r \tau)\,\mathrm{d}\tau\right\} \tag{3.25}$$

(see equation (3.7)), where $\boldsymbol{\psi} \equiv \psi_1 + \mathrm{i}\psi_2$ satisfies

$$\boldsymbol{\psi} = [\Omega^2 \Delta \boldsymbol{I} - \mathrm{i}\Omega\Delta\dot{\boldsymbol{I}} + \Omega \boldsymbol{h} - \mathrm{i}\dot{\boldsymbol{h}} + \mathrm{i}\hat{\boldsymbol{L}}]/\Omega^2(C-A) \tag{3.26}$$

if $\Delta \boldsymbol{I} \equiv \Delta I_{13} + \mathrm{i}\Delta I_{23}, \quad \boldsymbol{h} \equiv h_1 + \mathrm{i}h_2$

and
$$\hat{\boldsymbol{L}} \equiv \hat{L}_1 + \mathrm{i}\hat{L}_2. \tag{3.27}$$

The corresponding solution of equation (3.20) for the axial component is

$$m_3(t) = \psi_3(t) + \text{constant}. \tag{3.28}$$

In the absence of external torques (i.e. when $\hat{L}_i = 0$) equations (3.23) and (3.27) with appropriate values of $\boldsymbol{\psi}$ and ψ_3 give $\boldsymbol{m}$ and m_3 for the case when the total angular momentum of the whole Earth is conserved. Conservation of total angular momentum about the x_3 axis, as expressed by equation (3.28) with $\hat{L}_3 = 0$, gives

$$\Omega[C^{(c)} + C^{(m)}](1+m_3) + \Omega[\Delta I_{33}^{(c)} + \Delta I_{33}^{(m)} + \Delta I_{33}^{(a)}] + h_3^{(c)} + h_3^{(a)} = \text{constant}, \tag{3.29}$$

with corresponding but rather more complicated expressions for $\boldsymbol{m}$. Write

$$\boldsymbol{\psi} \equiv \boldsymbol{\psi}^{(c)} + \boldsymbol{\psi}^{(m)} + \boldsymbol{\psi}^{(a)} \quad \text{and} \quad \psi_3 \equiv \psi_3^{(c)} + \psi_3^{(m)} + \psi_3^{(a)}, \tag{3.30}$$

where $\boldsymbol{\psi}^{(c)}$ and $\psi_3^{(c)}$ comprise all the terms in $\boldsymbol{\psi}$ and ψ_3 respectively involving $\Delta I_{ij}^{(c)}, h_i^{(c)}$ etc., and likewise for $\boldsymbol{\psi}^{(m)}$, $\psi_3^{(m)}$, $\boldsymbol{\psi}^{(a)}$ and $\psi_3^{(a)}$. Equations (3.25) and (3.28) can then be written as follows

$$\boldsymbol{m}(t) = \exp(\mathrm{i}\sigma_r t)\left\{\boldsymbol{m}(0) - \mathrm{i}\sigma_r \int_0^t [\boldsymbol{\psi}^{(c)}(\tau) + \boldsymbol{\psi}^{(m)}(\tau) + \boldsymbol{\psi}^{(a)}(\tau)]\exp(-\mathrm{i}\sigma_r\tau)\,\mathrm{d}\tau\right\} \tag{3.31}$$

and
$$m_3(t) = \psi_3^{(c)}(t) + \psi_3^{(m)}(t) + \psi_3^{(a)}(t) + \text{constant} \tag{3.32}$$

respectively. As the angular momentum H_i of the whole Earth remains constant when $\hat{L}_i = 0$, changes in the angular velocity $\omega_i = \Omega(m_1, m_2, 1+m_3)$ of the solid Earth can only be produced by changes in the inertia tensor $I_{ij}^{(m)}$ and the exchange of angular momentum with the underlying core and the overlying hydrosphere and atmosphere.

So far as the evaluation of angular momentum exchange is concerned, the method to be adopted for determining the excitation functions $\psi_i^{(a)}$ and $\psi_i^{(c)}$ (and the contributions to $\psi_i^{(m)}$ that they produce because the solid Earth is not perfectly rigid) depends on the data available. In the so-called 'torque' approach, the rate of change of angular momentum $I_{ij}^{(m)}\omega_i$ of the solid Earth is directly related to the torques exerted upon its boundaries as a consequence of the motions within the fluid regions with which it is in contact. In the alternative 'angular momentum' approach, $\mathrm{d}(I_{ij}^{(m)}\omega_j)/\mathrm{d}t$ is taken to be equal and opposite (when $\hat{L}_i = 0$) to the rate of change of the total angular momentum of the fluid regions. Now the dominant contribution to $\psi_i^{(a)}$ comes from the tropospheric and lower stratospheric regions of the atmosphere, for which fairly abundant accurate wind and pressure observations are available, but surface stresses are not easy to evaluate. So the angular momentum rather than the torque approach is appropriate in the investigation of atmospheric excitation of changes in the Earth's rotation, and it is being used with great success in the study of short-term variations of all three components of ω_i of atmospheric origin and the determination by subtraction of non-meteorological contributions to $\dot{\omega}_i$ (for references see Hide 1984, 1985; Cazenave 1986; Dickey *et al.* 1986; Wahr 1988). However, for changes in ω_i brought about by the interaction between the core and the solid Earth, the angular momentum approach is not practicable because we have insufficient information about motions in the main body of the core (but see Jault *et al.* 1988). We are therefore obliged to use the less attractive torque approach, by evaluating stresses at the core–mantle boundary to the best of our ability.

Contributions to the time-series $\omega_i(t) = \Omega(m_1(t), m_2(t), 1+m_3(t))$ due to each of a wide variety of geophysical processes involved differ in their respective magnitudes and spectral characteristics. This together with the availability of daily values of $h_i^{(a)}(t)$ since the late 1970s is facilitating the determination of the slowly varying contribution

$$\tilde{\omega}_i = \Omega(\tilde{m}_1, \tilde{m}_2, 1+\tilde{m}_3) \qquad (3.33)$$

to $\omega_i(t)$, which is due largely to the action of torques at the core–mantle interface. By equation (3.20) we find

$$\Omega C^{(m)}\dot{\tilde{m}}_3 + \Omega[\Delta \dot{I}_{33}^{(m,m)} + \Delta \dot{I}_{33}^{(m,c)}] = L_3^*, \qquad (3.34)$$

where

$$L_3^* \equiv -(\Omega C^{(c)}\dot{\tilde{m}}_3 + \Omega \Delta \dot{I}_{33}^{(c)} + \dot{h}_3^{(c)}), \qquad (3.35)$$

L_3^* being the axial component of the torque L_i^* produced on the mantle by the core. The quantity $\Delta I_{33}^{(m,m)}$ is the $(i=3, j=3)$ component of the change of $\Delta I_{33}^{(m)}$ associated with processes within the solid Earth (e.g. earthquakes, melting of ice), and $\Delta I_{33}^{(m,c)}$ is the $(i=3, j=3)$ component of the contribution to $\Delta I_{ij}^{(m)}$ resulting from the deformation of the solid Earth produced by the forces responsible for core–mantle coupling. Such deformations might be important (cf. Hinderer *et al.* 1987; Merriam 1988), but to the accuracy of any direct determination of these deformations and of the core–mantle torque L_i^* we are likely to be able to make in the foreseeable future, they represent a correction no greater than the uncertainties in L_i^*. Thus we write

$$\dot{\tilde{m}}_3 \approx L_3^*/\Omega C^{(m)} \qquad (3.36)$$

as the leading approximation to the axial component of the equation of torque balance when 'atmospheric' and 'tidal' effects have been allowed for, at times when effects associated with $\Delta I_{ij}^{(m,m)}$ are negligible. This can be related to corresponding length-of-day changes through equation (3.8).

By equations (3.18) and (3.19), the leading approximations to the corresponding equatorial components of torque balance are the following:

$$A^{(m)}\Omega[\dot{\tilde{m}}_1 + \Omega A^{-1}\tilde{m}_2(C-A)] = L_1^*, \qquad (3.37a)$$

$$A^{(m)}\Omega[\dot{\tilde{m}}_2 - \Omega A^{-1}\tilde{m}_1(C-A)] = L_2^*. \qquad (3.37b)$$

When dealing with changes in $\tilde{m}_1$ and $\tilde{m}_2$ on timescales much greater than about a year, the terms involving $\dot{\tilde{m}}_1$ and $\dot{\tilde{m}}_2$ in the last two equations can be neglected, giving

$$(\tilde{m}_1, \tilde{m}_2) = [A/\Omega^2(C-A)A^{(m)}](-L_2^*, L_1^*). \qquad (3.38)$$

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4. Concluding remarks

The basic theoretical relation needed in the study of Earth rotation changes due to topographic torques at the core–mantle interface are given by equations (2.9), (3.36) and (3.38) or (3.37). The integrals on the right-hand side of equation (2.9) involve the core–mantle interface topography $h(\theta, \phi)$. When dealing with $L_1(t)$ and $L_2(t)$ and the polar motion $\tilde{m}_1(t)$ and $\tilde{m}_2(t)$ they produce, the dominant contribution to h is the equatorial bulge of the core–mantle interface, which corresponds to a 10 km difference between the equatorial radius and polar radius of the core. But the equatorial bulge makes no contribution to $L_3(t)$, which produces changes in the length of the day, so when dealing with such changes it is necessary to look in detail at features of h that depend on ϕ as well as θ. Over the past 20 years various attempts have been made to infer $h(\theta, \phi)$ from the pattern of long-wavelength gravity anomalies, with the most recent models incorporating the findings of seismic tomography and modern ideas about lower mantle rheology and convection (for references see papers by Hager & Richards and Woodhouse & Dziewonski, this Symposium).

The other quantity required in the evaluation of L_i from geophysical data is the field of horizontal velocity $\boldsymbol{u}_s = \boldsymbol{u}_s(\theta, \phi, t) = (0, v_s, w_s)$ in the free stream just below the core–mantle interface. Geomagnetic secular variation data have been used by various workers to infer $\boldsymbol{u}_s$ by a method based on the assumptions that: (*a*) on timescales very much less than that of free Ohmic decay of magnetic fields in the core, which is several thousand years for global-scale features, Alfvén's 'frozen magnetic flux' theorem can be applied, (*b*) in the outer reaches of the core the horizontal components of Coriolis forces are in geostrophic balance with the horizontal pressure gradient (see §2 above), and (*c*) the electrical conductivity of the mantle is everywhere very much less than that of the core; (for references see Whaler 1986; Jacobs 1987; Voorhies 1987; Bloxham 1988; Courtillot & Le Mouël 1988).

When Alfvén's theorem holds, the magnetic field $\boldsymbol{B}$ and the eulerian flow velocity $\boldsymbol{u}$ are related as follows:

$$\partial \boldsymbol{B}/\partial t = \nabla \times (\boldsymbol{u} \times \boldsymbol{B}). \tag{4.1}$$

Lines of force associated with long-wavelength features of the main geomagnetic field are advected by the horizontal flow $\boldsymbol{u}_s$ just below the core–mantle interface. Equation (4.1) above does not permit the unique determination of $\boldsymbol{u}_s$, but when the equation is combined with the geostrophic approximation expressed by equation (2.7) (see Backus & Le Mouël 1985; Voorhies 1987) it is possible in principle to determine $\boldsymbol{u}_s$ over most of the core, using as basic data values of $\boldsymbol{B}$ and $\partial \boldsymbol{B}/\partial t$ at the core–mantle interface, as obtained by the extrapolation of geomagnetic observations made at and near the Earth's surface. Various groups of geomagnetic workers are investigating the errors and uncertainties in the velocity fields so produced, which stem from imperfections in our knowledge of the spatial and temporal variations in the Earth's magnetic field and of the ranges of validity of the physical assumptions upon which the method is based.

A study of the application of equations (2.9), (3.36) and (3.38) making use of geophysical data being carried out in collaboration with R. W. Clayton, B. H. Hager, M. A. Spieth and C. V. Voorhies will be reported in a paper currently in preparation. The first results are encouraging, for they indicate that topographic coupling alone might account for the observed recent 'decade' changes in the Earth's rate of rotation without the necessity of invoking extreme models of $h(\theta, \phi)$ and $\boldsymbol{u}_s(\theta, \phi, t)$. Specifically, ϕ-variations in effective topographic height h of up to no more than about 0.5 km are implied by these calculations of topographic coupling. It might be significant that 0.5 km is also the magnitude of the departure from the equilibrium value of the equatorial bulge that Gwinn *et al.* (1986) (see also Wahr 1988) have inferred from their determinations of the amplitude and phase of free-core nutation based on very long baseline interferometry (VLBI) data. But 0.5 km is less by a factor of about 10 than the heights of irregular topography now being proposed by various workers on the basis of seismic tomography. This apparent discrepancy cannot be considered in detail here, but we note that the *effective* topographic height obtained by the method described in this paper should be the same as the actual height if the metallic core is in direct contact with the lower mantle. However, if there is a stable layer of poorly conducting low-viscosity liquid slag separating the metallic core from the solid mantle (D. L. Anderson, personal communication), then the pressure field acting on the actual topography could have weaker horizontal gradients than those present at the top of the metallic region, and the *effective* topographic height would in consequence be less than the actual height.

All of us involved with the investigation of the very difficult problems of determining the structure, composition and dynamics of the Earth's deep interior wish to see the development of suitable strategies for combining the wide variety of data available in the best possible way. It is to this end that the incomplete but in some ways novel material outlined in this paper is offered as a contribution to this Discussion Meeting on seismic tomography and mantle circulation.

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